AP Calculus AB
Substitution Class Notes

Name ______________________

Substitution Rule for Indefinite Integrals

If $u = g(x)$ is a differentiable function whose range is an interval I and f is continuous on I, then
$$\int f(g(x))g'(x)dx = \int f(u)du$$

1. $\int \sqrt{3x+2}dx$

$u = 3x+2$

$\frac{du}{dx} = 3$

$dx = \frac{du}{3}$

$\int u^{1/2} \cdot \frac{du}{3}$

$\int \frac{1}{3} u^{1/2} du$

$\frac{1}{3} u^{3/2} \cdot \frac{2}{3} + C$

$\frac{2}{9}(3x+2)^{3/2} + C$

Confirm your answer by taking the derivative:

$\frac{2}{9}(3x+2)^{3/2} + C$

$\frac{6}{18}(3x+2)^{1/2} \cdot 3$

$(3x+2)^{1/2}$

$\sqrt{3x+2}$

2. $\int \frac{x}{\sqrt{x^2+1}}dx$

$u = x^2+1$

$\frac{du}{dx} = 2x$

$dx = \frac{du}{2x}$

$\int \frac{x}{u^{1/2}} \cdot \frac{du}{2x}$

$\int \frac{1}{2} u^{-1/2} du$

$\frac{1}{2} u^{1/2} \cdot 2 + C$

$\sqrt{x^2+1} + C$

3. $\int x^2(x^3+4)^{10}dx$

$u = x^3+4$

$\frac{du}{dx} = 3x^2$

$dx = \frac{du}{3x^2}$

$\int \cancel{x^2} \cdot u^{10} \cdot \frac{du}{3\cancel{x^2}}$

$\int \frac{1}{3} u^{10} du$

$\frac{1}{33} u^{11} + C$

$\frac{1}{33}(x^3+4)^{11} + C$

4. $\int \frac{x+1}{(x^2+2x+7)^3}dx$

$u = x^2+2x+7$

$\frac{du}{dx} = 2x+2$

$dx = \frac{du}{2(x+1)}$

$\int \frac{\cancel{x+1}}{u^3} \cdot \frac{du}{2(\cancel{x+1})}$

$\int \frac{1}{2} u^{-3} du$

$\frac{1}{2} u^{-2} \cdot \frac{1}{-2} + C$

$-\frac{1}{4}(x^2+2x+7)^{-2} + C$

5. $\int x\sin x^2 dx$

$u = x^2$

$\frac{du}{dx} = 2x$

$dx = \frac{du}{2x}$

$\int \cancel{x}\sin u \cdot \frac{du}{2\cancel{x}}$

$\int \frac{1}{2}\sin u\, du$

$-\frac{1}{2}\cos x^2 + C$

6. $\int \cos 3x \sin^3 3x dx$

$u = \sin 3x$

$\frac{du}{dx} = \cos 3x \cdot 3$

$dx = \frac{du}{3\cos 3x}$

$\int \cancel{\cos 3x}\, u^3 \cdot \frac{du}{3\cancel{\cos 3x}}$

$\int \frac{1}{3} u^3 du$

$\frac{1}{12} u^4 + C$

$\frac{1}{12} \sin^4 3x + C$

7. $\int \frac{\sec^2\left(\frac{1}{x}\right)}{x^2} dx$

$u = x^{-1}$

$\frac{du}{dx} = -x^{-2}$

$\frac{du}{dx} = \frac{-1}{x^2}$

$x^2 du = -dx$

$dx = -x^2 du$

$\int \frac{\sec^2 u}{\cancel{x^2}} \cdot -\cancel{x^2} du$

$\int -\sec^2 u \, du$

$-\tan\left(\frac{1}{x}\right) + C$

8. $\int \frac{\sec^2\left(\frac{1}{x}\right)\tan^5\left(\frac{1}{x}\right)}{x^2} dx$

$u = \tan \frac{1}{x}$

$\frac{du}{dx} = \sec^2 \frac{1}{x} \cdot -x^{-2}$

$\frac{du}{dx} = \frac{-\sec^2 \frac{1}{x}}{x^2}$

$x^2 du = -\sec^2 \frac{1}{x} dx$

$dx = \frac{x^2 du}{-\sec^2 \frac{1}{x}}$

$\int \frac{\cancel{\sec^2}\left(\frac{1}{x}\right) \cdot u^5}{\cancel{x^2}} \cdot \frac{\cancel{x^2} du}{-\cancel{\sec^2}\left(\frac{1}{x}\right)}$

$\int -u^5 du$

$-\frac{1}{6} u^6 + C$

$-\frac{1}{6} \tan^6\left(\frac{1}{x}\right) + C$